

CHAPTER - 5

PLAYING WITH NUMBERS

EXERCISE 5(A)

Question 1.

Write the quotient when the sum of 73 and 37 is divided by

- (i) 11
- (ii) 10

Solution:

Sum of 73 and 37 is to be divided by

- (i) 11
- (ii) 10

Let $ab = 73$

and $ba = 37$

$\therefore a = 7$

and $b = 3$

- (i) The quotient of $ab + ba$ i.e. $(73 + 37)$ when divided by 11 is $a + b = 7 + 3 = 10$

$$\left(\because \frac{ab + ba}{11} = a + b \right)$$

- (ii) The quotient of $ab + ba$ i.e. $(73 + 37)$ when divided by 10 (i.e. $a + b$) is 11

$$\left(\because \frac{ab + ba}{a + b} = 11 \right)$$

Question 2.

Write the quotient when the sum of 94 and 49 is divided by

- (i) 11
- (ii) 13

Solution:

Sum of 94 and 49 is to be divided by

- (i) 11 (ii) 13

Let $ab = 94$

and $ba = 49$

$\therefore a = 9$ and $b = 4$

- (i) The quotient of $94 + 49$ (i.e. $ab + ba$)

When divided by 11 is $a + b$ i.e. $9 + 4 = 13$

$$\left(\because \frac{ab + ba}{11} = a + b \right)$$

- (ii) The quotient of $94 + 49$ (i.e. $ab + ba$)

When divided by 13 i.e. $(a + b)$ is 11

$$\left(\because \frac{ab + ba}{a + b} = 11 \right)$$

Question 3.

Find the quotient when $73 - 37$ is divided by

(i) 9

(ii) 4

Solution:

Difference of $73 - 37$ is to be divided by

- (i) 9 (ii) 4

Let $ab = 73$ and $ba = 37$

$\therefore a = 7$ and $b = 3$

- (i) The quotient of $73 - 37$ (i.e. $ab - ba$) when divided by 9 is $a - b$ i.e. $7 - 3 = 4$

$$\left(\because \frac{ab - ba}{9} = a - b \right)$$

- (ii) The quotient of $73 - 37$ (i.e. $ab - ba$) when divided by 4 i.e. $(a - b)$ is 9

$$\left(\because \frac{ab - ba}{a - b} = 9 \right)$$

Question 4.

Find the quotient when $94 - 49$ is divided by

- (i) 9
- (ii) 5

Solution:

Difference of 94 and 49 is to be divided by

- (i) 9
- (ii) 5

Let $ab = 94$ and $ba = 49$

$\therefore a = 9$ and $b = 4$

- (i) The quotient of $94 - 49$ i.e. $(ab - ba)$ when divided by 9 is $(a - b)$ i.e. $9 - 4 = 5$

$$\left(\because \frac{ab - ba}{9} = a - b \right)$$

- (ii) The quotient of $94 - 49$ i.e. $(ab - ba)$ when divided by 5 i.e. $(a - b)$ is 9

$$\left(\because \frac{ab - ba}{a - b} = 9 \right)$$

Question 5.

Show that $527 + 752 + 275$ is exactly divisible by 14.

Solution:

Property :

$abc = 100a + 10b + c$ (i)

$bca = 100b + 10c + a$ (ii)

and $cab = 100c + 10a + b$ (iii)

Adding, (i), (ii) and (iii), we get $abc + bca + cab = 111a + 111b + 111c = 111(a + b + c) = 3 \times 37(a + b + c)$

Now, let us try this method on

$527 + 752 + 275$ to check is it exactly divisible by 14

Here, $a = 5, b = 2, c = 7$

$527 + 752 + 275 = 3 \times 37(5 + 2 + 7) = 3 \times 37 \times 14$

Hence, it shows that $527 + 752 + 275$ is exactly divisible by 14

Question 6.

If $a = 6$, show that $abc = bac$.

Solution:

Given : $a = 6$

To show : $abc = bac$

Proof: $abc = 100a + 10b + c$ (i)

(By using property 3)

$bac = 100b + 10a + c$ —(ii)

(By using property 3)

Since, $a = 6$

Substitute the value of $a = 6$ in equation (i) and (ii), we get

$$abc = 1006 + 106 + c \dots\dots\dots(iii)$$

$$bac = 1006 + 106 + c \dots\dots\dots(iv)$$

Subtracting (iv) from (iii) $abc - bac = 0$

$$abc = bac$$

Hence proved.

Question 7.

If $a > c$; show that $abc - cba = 99(a - c)$.

Solution:

Given, $a > c$

To show : $abc - cba = 99(a - c)$

Proof: $abc = 100a + 10b + c \dots\dots\dots(i)$

(By using property 3)

$$cba = 100c + 10b + a \dots\dots\dots(ii)$$

(By using property 3)

Subtracting, equation (ii) from (i), we get

$$abc - cba = 100a + c - 100c - a$$

$$abc - cba = 99a - 99c$$

$$abc - cba = 99(a - c)$$

Hence proved.

Question 8.

If $c > a$; show that $cba - abc = 99(c - a)$.

Solution:

Given : $c > a$

To show : $cba - abc = 99(c - a)$

Proof:

$$cba = 100c + 10b + a \dots\dots\dots(i)$$

(By using property 3)

$$abc = 100a + 10b + c \dots\dots\dots(ii)$$

(By using property 3)

Subtracting (ii) from (i)

$$cba - abc = 100c + 10b + a - 100a - 10b - c$$

$$\Rightarrow cba - abc = 99c - 99a$$

$$\Rightarrow cba - abc = 99(c - a)$$

Hence proved.

Question 9.

If $a = c$, show that $cba - abc = 0$.

Solution:

Given : $a = c$

To show : $cba - abc = 0$

Proof:

$$cba = 100c + 10b + a \dots\dots\dots(i)$$

(By using property 3)

$$abc = 100a + 10b + c \dots\dots\dots(ii)$$

(By using property 3)

Since, $a = c$,

Substitute the value of $a = c$ in equation (i) and (ii), we get

$$cba = 100c + 10b + c \dots\dots\dots(iii)$$

$$abc = 100c + 10b + c \dots\dots\dots(iv)$$

Subtracting (iv) from (iii), we get

$$cba - abc = 100c + 10b + c - 100c - 10b - c$$

$$\Rightarrow cba - abc = 0$$

$$\Rightarrow cba = abc$$

Hence proved.

Question 10.

Show that $954 - 459$ is exactly divisible by 99.

Solution:

To show : $954 - 459$ is exactly divisible by 99, where $a = 9$, $b = 5$, $c = 4$

$$abc = 100a + 10b + c$$

$$\Rightarrow 954 = 100 \times 9 + 10 \times 5 + 4$$

$$\Rightarrow 954 = 900 + 50 + 4 \dots\dots\dots(i)$$

$$\text{and } 459 = 100 \times 4 + 10 \times 5 + 9$$

$$\Rightarrow 459 = 400 + 50 + 9 \dots\dots\dots(ii)$$

Subtracting (ii) from (i), we get

$$954 - 459 = 900 + 50 + 4 - 400 - 50 - 9$$

$$\Rightarrow 954 - 459 = 500 - 5$$

$$\Rightarrow 954 - 459 = 495$$

$$\Rightarrow 954 - 459 = 99 \times 5$$

Hence, $954 - 459$ is exactly divisible by 99

Hence proved.

EXERCISE 5(B)

Question 1.

$$\begin{array}{r} 3A \\ + 25 \\ \hline B2 \end{array}$$

Solution:

$A = 7$ as $7 + 5 = 12$. We want 2 at units place and 1 is carry over. Now $3 + 2 + 1 = 6$.

$$B = 6$$

Hence $A = 7$ and $B = 6$

$$\begin{array}{r} 37 \\ + 25 \\ \hline 62 \end{array}$$

Question 2.

$$\begin{array}{r} 98 \\ + 4A \\ \hline CB3 \end{array}$$

Solution:

$A = 5$ as $8 + 5 = 13$. We want 3 at units place and 1 is carry over. Now $9 + 4 + 1 = 14$.

$$B = 4 \text{ and } C = 1$$

Hence $A = 5$ and $B = 4$ and $C = 1$

$$\begin{array}{r} 98 \\ + 45 \\ \hline 143 \end{array}$$

Question 3.

$$\begin{array}{r} A1 \\ + 1B \\ \hline B0 \end{array}$$

Solution:

$B = 9$ as $9 + 1 = 10$. We want 0 at units place and 1 is carry over. Now $B - 1 - 1 = A$.

$$\therefore A = 9 - 2 = 7$$

Hence $A = 7$ and $B = 9$

$$\begin{array}{r} 71 \\ + 19 \\ \hline 90 \end{array}$$

Question 4.

$$\begin{array}{r} 2AB \\ +AB1 \\ \hline B18 \end{array}$$

Solution:

$B = 7$ as $7 + 1 = 8$. We want 8 at unit place.

Now

$$7 + A = 11$$

$$\therefore A = 11 - 7 = 4$$

Hence $A = 4$ and $B = 7$

$$\begin{array}{r} 247 \\ +471 \\ \hline 718 \end{array}$$

Question 5.

$$\begin{array}{r} 12A \\ +6AB \\ \hline A09 \end{array}$$

Solution:

$$A + B = 9$$

$$\text{and } 2 + A = 10$$

$$\therefore A = 10 - 2 = 8$$

$$\text{and } 8 + B = 9$$

$$\therefore B = 9 - 8 = 1$$

Hence $A = 8$ and $B = 1$

$$\begin{array}{r} 128 \\ +681 \\ \hline 809 \end{array}$$

Question 6.

$$\begin{array}{r} 1A \\ \times A \\ \hline 9A \end{array}$$

Solution:

As we need A at unit place and 9 at ten's place,

$A = 6$ as $6 \times 6 = 36$

$$\begin{array}{r} 16 \\ \times 6 \\ \hline 96 \end{array}$$

Question 7.

$$\begin{array}{r} AB \\ \times 6 \\ \hline BBB \end{array}$$

Solution:

As we need B at unit place and B at ten's place,

$$\therefore B = 4 \text{ as } 6 \times 4 = 24$$

Now we want to find A, $6 \times A + 2 = 4$ (at unit's place)

$$\therefore A = 7$$

$$\begin{array}{r} 74 \\ \times 6 \\ \hline 444 \end{array}$$

Question 8.

$$\begin{array}{r} AB \\ \times 3 \\ \hline CAB \end{array}$$

Solution:

As we need B at unit place and A at ten's place,

$$\therefore B = 0 \text{ as } 3 \times 0 = 0$$

Now we want to find A, $3 \times A = A$ (at unit's place)

$$\therefore A = 5, \text{ as } 3 \times 5 = 15$$

$$\therefore C = 1$$

$$\begin{array}{r} 50 \\ \times 3 \\ \hline 150 \end{array}$$

Question 9.

$$\begin{array}{r} AB \\ \times 5 \\ \hline CAB \end{array}$$

Solution:

As we need B at unit place and A at ten's place,

$$B = 0 \text{ as } 5 \times 0 = 0$$

Now we want to find A, $5 \times A = A$ (at unit's place)

$$A = 5, \text{ as } 5 \times 5 = 25$$

$$C = 2$$

$$\begin{array}{r} 50 \\ \times 5 \\ \hline 250 \end{array}$$

Question 10.

$$\begin{array}{r} 8A5 \\ +94A \\ \hline 1A33 \end{array}$$

Solution:

$$5 + A = 13$$

$$\text{and } A + 4 = 13$$

$$\therefore A = 13 - 5 = 8$$

$$\text{Hence } A = 8$$

$$\begin{array}{r} 885 \\ +948 \\ \hline 1833 \end{array}$$

Question 11.

$$\begin{array}{r} 6AB5 \\ +D58C \\ \hline 9351 \end{array}$$

Solution:

$$C + 5 = 11$$

$$\therefore C = 11 - 5 = 6$$

$$\text{and } 8 + B + 1 = 15$$

$$\therefore B = 15 - 9 = 6$$

$$\text{and } A + 5 + 1 = 13$$

$$\therefore A = 13 - 6 = 7$$

$$\text{and } 6 + D + 1 = 9$$

$$\therefore D = 9 - 7 = 2$$

Hence $A = 7$, $B = 6$, $C = 6$ and $D = 2$

$$\begin{array}{r} 6765 \\ + 2586 \\ \hline 9351 \end{array}$$

EXERCISE 5(C)

Question 1.

Find which of the following numbers are divisible by 2:

(i) 192

(ii) 1660

(iii) 1101

(iv) 2079

Solution:

A number having its unit digit 2,4,6,8 or 0 is divisible by 2,

So, Number 192, 1660 are divisible by 2.

Question 2.

Find which of the following numbers are divisible by 3:

(i) 261

(ii) 111

(iii) 6657

(iv) 2574

Solution:

A number is divisible by 3 if the sum of its digits is divisible by 3,

So, 261, 111 are divisible by 3.

Question 3.

Find which of the following numbers are divisible by 4:

(i) 360

(ii) 3180

(iii) 5348

(iv) 7756

Solution:

A number is divisible by 4, if the number formed by the last two digits is divisible by 4.
So, Number 360, 5348, 7756 are divisible by 4.

Question 4.

Find which of the following numbers are divisible by 5 :

(i) 3250

(ii) 5557

(iii) 39255

(iv) 8204

Solution:

A number having its unit digit 5 or 0, is divisible by 5.
So, 3250, 39255 are all divisible by 5.

Question 5.

Find which of the following numbers are divisible by 10:

(i) 5100

(ii) 4612

(iii) 3400

(iv) 8399

Solution:

A number having its unit digit 0, is divisible by 10.
So, 5100, 3400 are all divisible by 10.

Question 6.

Which of the following numbers are divisible by 11 :

(i) 2563

(ii) 8307

(iii) 95635

Solution:

A number is divisible by 11 if the difference of the sum of digits at the odd places and sum of the digits at even places is zero or divisible by 11.

So, 2563 is divisible by 11.

EXERCISE 5(D)

For what value of digit x, is :

Question 1.

1x5 divisible by 3 ?

Solution:

1x5 is divisible by 3

$\Rightarrow 1 + x + 5$ is a multiple of 3

$\Rightarrow 6 + x = 0, 3, 6, 9,$

$\Rightarrow x = -6, -3, 0, 3, 6, 9$
Since, x is a digit
 $x = 0, 3, 6$ or 9

Question 2.

$31x5$ divisible by 3 ?

Solution:

$31x5$ is divisible by 3
 $\Rightarrow 3 + 1 + x + 5$ is a multiple of 3
 $\Rightarrow 9 + x = 0, 3, 6, 9,$
 $\Rightarrow x = -9, -6, -3, 0, 3, 6, 9,$
Since, x is a digit
 $x = 0, 3, 6$ or 9

Question 3.

$28x6$ a multiple of 3 ?

Solution:

$28x6$ is a multiple of 3
 $2 + 8 + x + 6$ is a multiple of 3
 $\Rightarrow 16 + x = 0, 3, 6, 9, 12, 15, 18$
 $\Rightarrow x = -18, -5, -2, 0, 2, 5, 8$
Since, x is a digit = $2, 5, 8$

Question 4.

$24x$ divisible by 6 ?

Solution:

$24x$ is divisible by 6
 $\Rightarrow 2 + 4 + x$ is a multiple of 6
 $\Rightarrow 6 + x = 0, 6, 12$
 $\Rightarrow x = -6, 0, 6$
Since, x is a digit
 $x = 0, 6$

Question 5.

$3x26$ a multiple of 6 ?

Solution:

$3x26$ is a multiple of 6
 $3 + x + 2 + 6$ is a multiple of 3
 $\Rightarrow 11 + x = 0, 3, 6, 9, 12, 15, 18, 21,$
 $\Rightarrow x = -11, -8, -5, -2, 1, 4, 7, 10, \dots$
Since, x is a digit
 $x = 1, 4$ or 7

Question 6.

42x8 divisible by 4 ?

Solution:

42x8 is divisible by 4

$\Rightarrow 4 + 2 + x + 8$ is a multiple of 2

$\Rightarrow 14 + x = 0, 2, 4, 6, 8,$

$\Rightarrow x = -8, -6, -4, -2, 2, 4, 6, 8,$

Since, x is a digit 2, 4, 6, 8

Question 7.

9142x a multiple of 4 ?

Solution:

9142x is multiple of 4

$\Rightarrow 9 + 1 + 4 + 2 + x$ is a multiple of 4

$\Rightarrow 16 + x = 0, 4, 8, \dots\dots\dots$

$x = -8, -4, 0, 4, 8$

Since, x is a digit

4, 8

Question 8.

7x34 divisible by 9 ?

Solution:

7x34 is multiple of 9

$\Rightarrow 7 + x + 3 + 4$ is a multiple of 9

$\Rightarrow 14 + x = 0, 9, 18, 27,$

$\Rightarrow x = -1, 4, 13,$

Since, x is a digit

$x = 4$

Question 9.

5x555 a multiple of 9 ?

Solution:

Sum of the digits of 5x555

$= 5 + x + 5 + 5 + 5 = 20 + x$

It is multiple by 9

The sum should be divisible by 9

Value of x will be 7

Question 10.

3x2 divisible by 11 ?

Solution:

Sum of the digit in even place = x

and sum of the digits in odd place = $3 + 2 = 5$

Difference of the sum of the digits in even places and in odd places = $x - 5$

3x2 is a multiple of 11

$$\Rightarrow x - 5 = 0, 11, 22,$$

$$\Rightarrow x = 5, 16, 27,$$

Since, x is a digit $x = 5$

Question 11.

$5x2$ a multiple of 11 ?

Solution:

Sum of a digit in even place = x

and sum of the digits in odd place = $5 + 2 = 7$

Difference of the sum of the digits in even places and in odd places = $x - 7$

$5x2$ is a multiple of 11

$$\Rightarrow x - 7 = 0, 11, 22,$$

$$\Rightarrow x = 7, 18, 29,$$

Since, x is a digit

$$x = 7$$